KTH ROYAL INSTITUTE
OF TECHNOLOGY

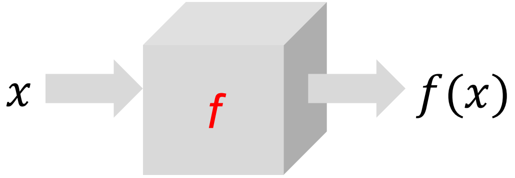
Functions

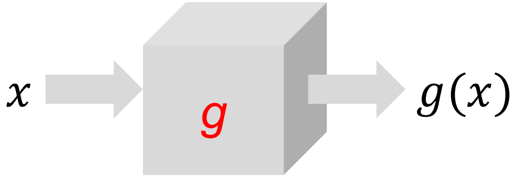
Composite functions, The inverse function,

Quadratic functions (Standard form, Vertex form, Factorised form)

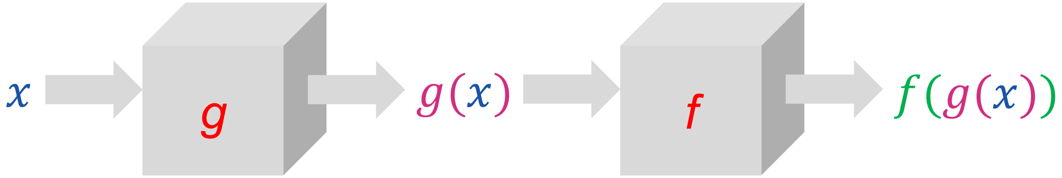
Hiroki Tomita

Composite functions





$$(f \circ g)(x) = f(g(x))$$



Ex.) $f(x) = x^2 + 2,\ \ g(x) = x + 4$
What is $(f \circ g)(x)$ and $(g \circ f)(x)$?

Ans.

$$(f \circ g)(x) = x^2 + 8x + 18$$

$$(g \circ f)(x) = x^2 + 6$$

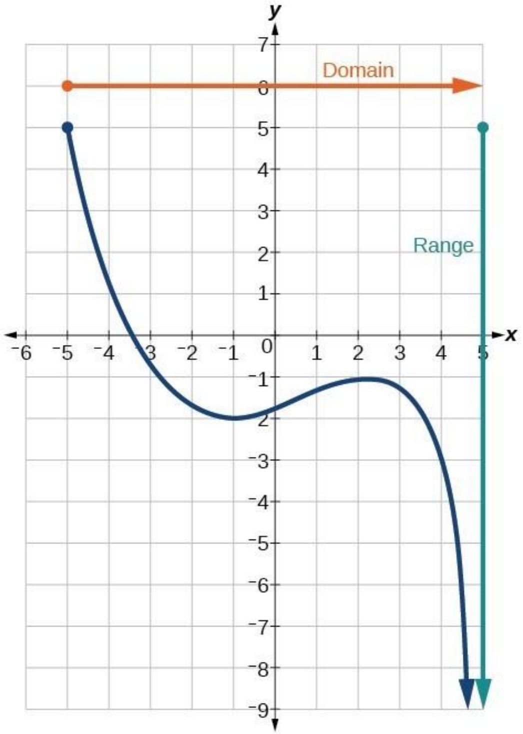
Domain and Range

- Domain

  The set of possible input values, shown on the x-axis

- Range

  The set of possible output values, shown on the y-axis



Domain and Range (of composite functions)

Ex.) $f(x) = x^2 + 2,\ \ g(x) = \sqrt{x-3}$

1. What is $(f \circ g)(x)$ and $(g \circ f)(x)$?

Ans.

$$(f \circ g)(x) = x - 1$$
$$(g \circ f)(x) = \sqrt{x^2 - 1}$$

2. What is the domain and range for $f(x)$ and $g(x)$?

Ans.

$$f(x)\colon -\infty < x < \infty, y \geq 2$$
$$g(x)\colon x \geq 3, y \geq 0$$

Inside of the square root is always greater than or equal to 0

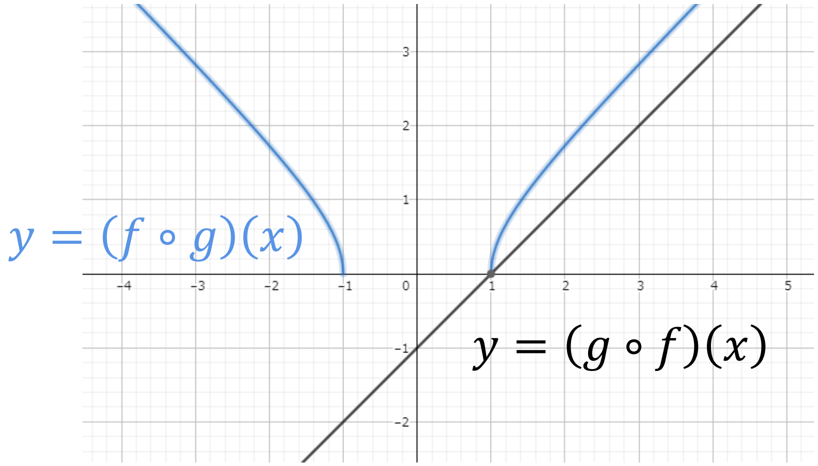
≥ 0

Domain and Range (of composite functions)

Ex.) $f(x) = x^2 + 2,\ g(x) = \sqrt{x-3}$

3. What is the domain and range for $(f \circ g)(x)$ and $(g \circ f)(x)$?

$$(f \circ g)(x) = x - 1$$
$$(g \circ f)(x) = \sqrt{x^2 - 1}$$

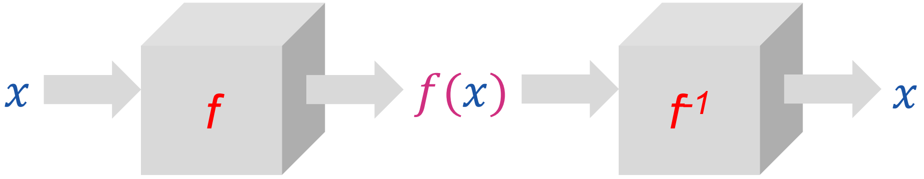


Ans.

$f(x)$: $-\infty < x < \infty,\qquad -\infty < y < \infty$

$g(x)$: $x \leq -1, 1 \leq x,\qquad y \geq 0$

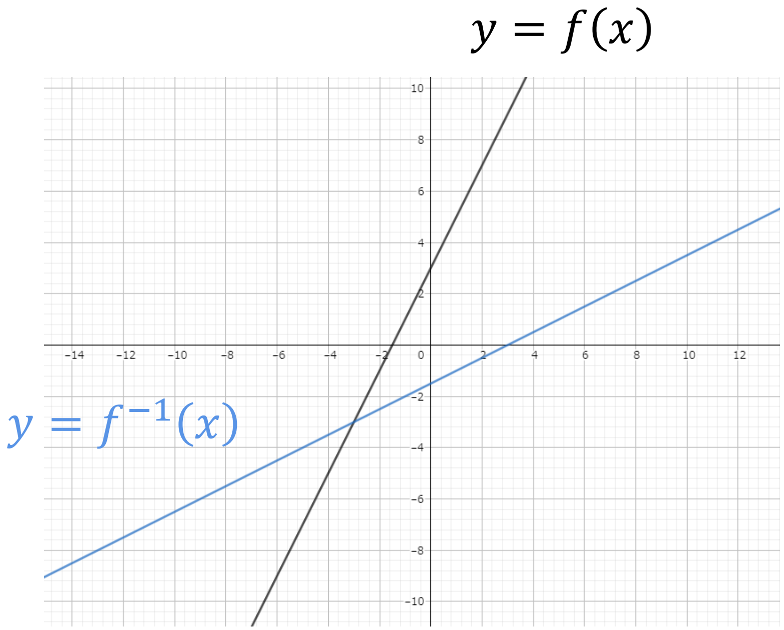
The inverse function



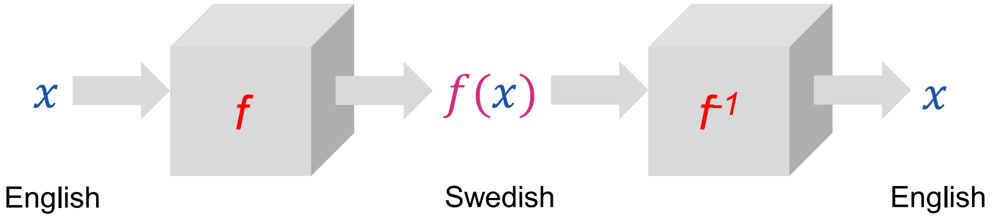
Ex.) $f(x) = 2x + 3$
What is $f^{-1}(x)$?

Ans. $f^{-1}(x) = \frac{x-3}{2}$

Does every function have the inverse functions?



Example (to understand the line test)



$f(x)$: translation from Eng to Swe
$f^{-1}(x)$: translation from Swe to Eng

one → ett → one

Very good → Mycket bra → Very good

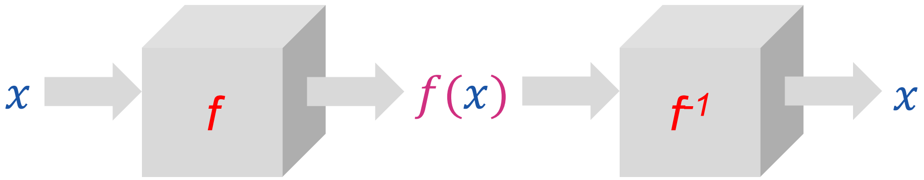
Hmm… It's working as the inverse function…

married → gift → …poison… or married…?

He encounters her → Han stöter på henne → …He hangs out with her… or he encounters her…?

Two possible answers!!

The inverse function (line test)



Ex.) $f(x) = x^2$

$f^{-1}(x) = \sqrt{x}$? or $f^{-1}(x) = -\sqrt{x}$?

There is no inverse function!!!

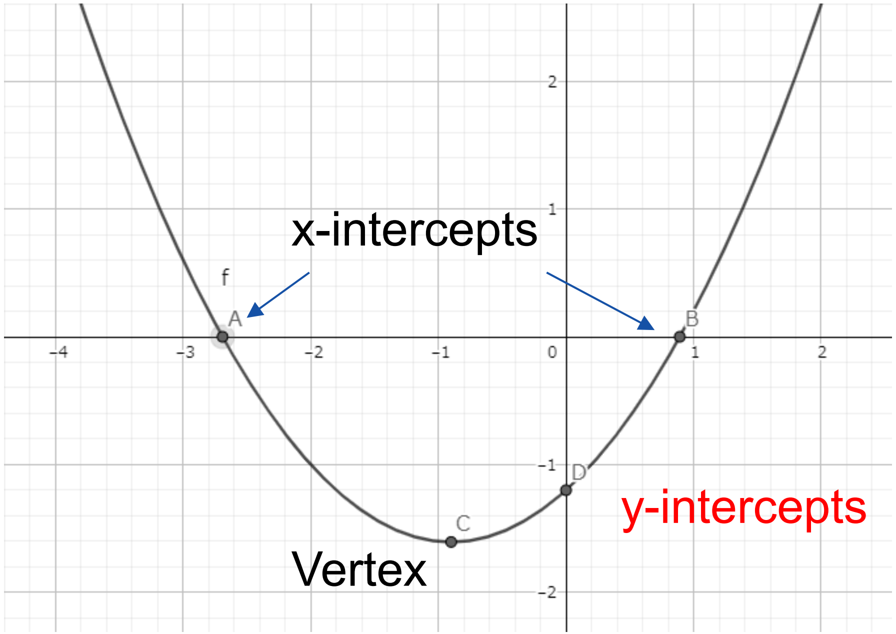
Quadratic functions

$$y = f(x) = ax^2 + bx + c$$

In this standard form, y-intercept is easily calculated

Y-intercept is the y value when x=0

Y-intercept is (0,c)



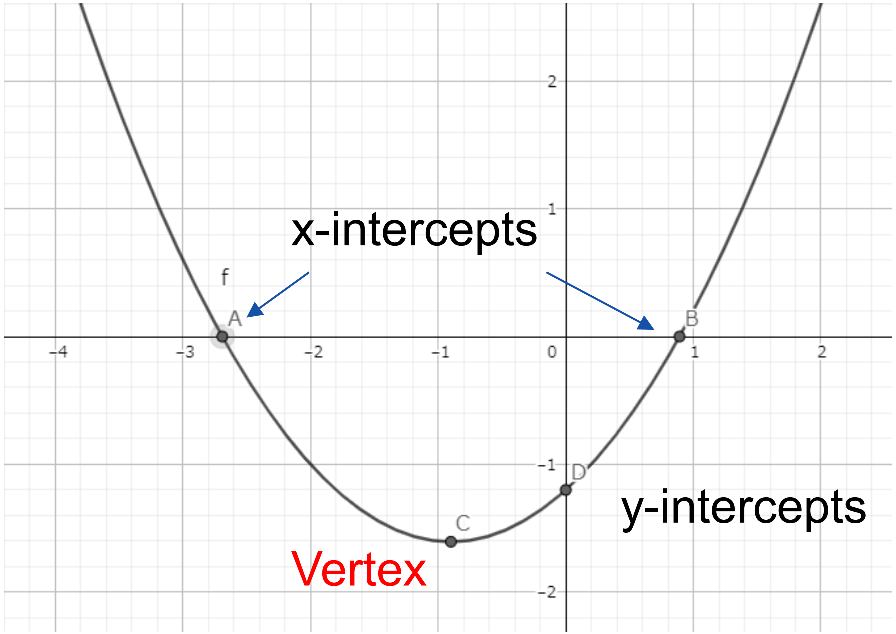
Vertex form of quadratic functions

$$y = f(x) = a(x - h)^2 + k$$

In this standard form, vertex is easily calculated

When $a > 0$, $(x - h)^2$ is always positive but 0 when $x = h$
The y value is minimum at $x = h$
(when $a < 0$, maximum at $x = h$)

Vertex is (h,k)



Completing the square

Method to convert standard form to vertex form

$$(x+a)^2 = x^2 + 2ax + a^2$$

Use the opposite direction of this equation to make $(x+a)^2$ form

Ex.) $f(x) = 2x^2 + 6x + 1$

$$f(x) = 2x^2 + 6x + 1$$
$$= 2(x^2 + 3x) + 1$$
$$= 2\left(x^2 + 2 \times \frac{3}{2}x + \frac{9}{4} - \frac{9}{4}\right) + 1$$
$$= 2\left(x + \frac{3}{2}\right)^2 - \frac{9}{2} + 1$$
$$= 2\left(x + \frac{3}{2}\right)^2 - \frac{7}{2}$$

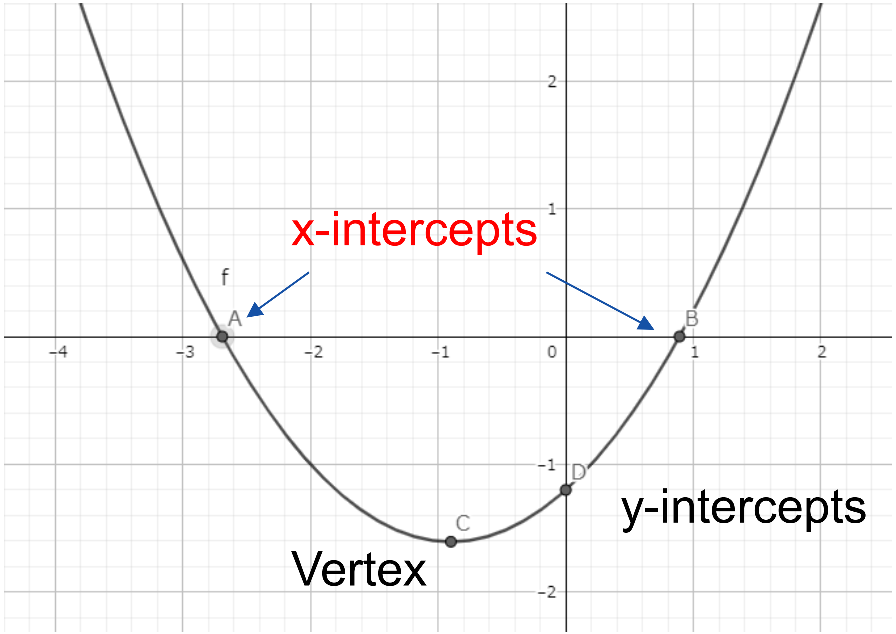
Factorised form of quadratic functions

$$y = f(x) = a(x - p)(x - q)$$

In this standard form, x-intercepts are easily calculated

When $x = p, q$, the y value is 0

X-intercepts are (p,0), (q,0)



Find the function from x-intercepts

Ex.) x-intercepts are (1,0) and (5,0) and passes through (-1, -8)
Find the function

Ans.
The function can be written as $f(x) = a(x-1)(x-5)$
It passes through (-1,-8), so $-8 = a(-1-1)(-1-5) = 12a$

$$a = -\frac{2}{3}$$

$$f(x) = -\frac{2}{3}(x-1)(x-5)$$

$$= -\frac{2}{3}x^2 + 4x - \frac{10}{3}$$